Scaling Properties of Delay Tolerant Networks with Correlated Motion Patterns[*]

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ABSTRACT

Mobile wireless networks with intermittent connectivity, often called Delay/Disruption Tolerant Networks (DTNs), have recently received a lot of attention because of their utility in various application scenarios where delay is noncritical. DTN routing and transport protocols effectively overcome partial connectivity by letting the nodes carry-and-forward data. The scalability of DTN protocols is very important for protocol design and evaluation. In particular, we need models that allow us to predict the performance of DTNs as a function of node mobility behavior (e.g., inter-contact times). Yet so far little work has been done to develop a unified framework that formalizes DTN performance as a function of motion behavior. In this paper, we represent DTNs as a class of wireless mobile networks with intermittent connectivity, where the inter-contact behavior of an arbitrary pair of nodes can be described by a generalized two-phase distribution consisting of a power-law head with an exponential tail, which represents correlated node mobility. Recent experiments have confirmed that such a two-phase distribution is a more realistic model for real traces collected from vehicular and pedestrian scenarios than the previous models based on random mobility and Poisson assumptions. Using this DTN model, we make the following contributions. First, we extend the throughput and delay scaling results of Grossglauser and Tse (originally derived for an exponential inter-contact time distribution) to a more general mobility model with a two-phase distribution. Second, we analyze the impact of finite buffer on the capacity scaling properties

of DTNs, again for different correlation behaviors. Finally, we validate our analytical results with a simulation study.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Wireless communication; C.4 [**Performance of Systems**]: Modeling techniques

General Terms

Theory, Performance

Keywords

Delay tolerant networks, capacity and delay analysis, correlated motion patterns

1. INTRODUCTION

Mobile wireless networks that can withstand intermittent connectivity, often called Delay/Disruption Tolerant Networks (DTNs), are becoming increasingly popular because of their applicability to various scenarios ranging from inter-vehicle communications [1, 19] to content distribution in challenged networks [20, 14]. DTNs have been also used to improve throughput at the expense of increased delays. It has been shown that DTN routing and transport protocols can benefit from node mobility and overcome the capacity bound of $\Theta(1/\sqrt{n \log n})$[1] originally established by Gupta and Kumar [12] for a fixed connected wireless network at the expense of increased delay. Noting that the average hop length of a path is the key limiting factor, Grossglauser and Tse [11] proposed a two-hop relay routing algorithm that exploits node mobility to effectively reduce the hop length, and utilizes relay nodes to deliver data to the destination when they meet.

[1]Here, n is the number of nodes. Recall that (i) $f(n) = O(g(n))$ means that $\exists c$ and $\exists N$ such that $f(n) \leq cg(n)$ for $n > N$ (i.e., asymptotic upper bound); (ii) $f(n) = \Omega(g(n))$ means that $\exists c$ and $\exists N$ such that $f(n) \geq cg(n)$ for $n > N$ (i.e., asymptotic lower bound); (iii) $f(n) = \Theta(g(n))$ means that $f(n) \in O(g(n)) \cap \Omega(g(n))$ (i.e., asymptotic tight bound); (iv) $f(n) = o(g(n))$ means that $\lim_{n \to \infty} f(n)/g(n) = 0$ (i.e., asymptotic insignificance); and (v) $f(n) = \omega(g(n))$ means $\lim_{n \to \infty} f(n)/g(n) = \infty$ (i.e., asymptotic dominance).

This result has been followed by a flurry of research activities that tried to characterize the delay/capacity relationship with respect to node mobility. Various mobility models have been considered, from a simple independent and identically distributed (I.I.D.) mobility model [24, 22, 31], to more complex random mobility models, such as random waypoint [29], random direction [28], uniform mobility [5], Brownian mobility [21], and random walk [6]. Sharma et al. [28] studied the impact of different mobility models on delay/capacity trade-offs. Garetto et al. [8] studied a home-point mobility model where each node moves around its home-point, and studied its impact on capacity scaling properties. In addition, Herdtner et al. [13] studied the impact of finite buffer on the capacity of a network; and Lee et al. [18] studied improving the throughput bound of *wireless multicast* by extending the two-hop relay algorithm to the multicast scenario.

Despite the wealth of analytic results about the capacity scaling properties and related trade-offs of wireless networks under various constraints, our understanding of the basic scaling properties of DTNs is still limited and fragmented. This is in part due to the fact that so far little work has been done to study DTNs in a unified framework that can explain the fundamental characteristics of the throughput/delay scaling properties of a delay tolerant network.

In this paper, we start by representing DTNs as a class of wireless mobile networks with intermittent connectivity, where the inter-contact times of an arbitrary pair of nodes correspond to a homogeneous Poisson process (i.e., exponential pairwise inter-contact time). Groenevelt et al. [10] showed that *random* mobility models such as random direction can be modeled using an exponential inter-contact time distribution where the pairwise inter-contact rate of arbitrary nodes results to be proportional to *radio range* and *node speed*. We then extend the model to accommodate two-phase distributions with a power-law head and an exponential tail.

When the radio range scales below the critical connectivity threshold $\Theta(\sqrt{\log n / n})$, the network becomes disconnected with high probability [12]. Under such circumstances, we note that radio range and node speed mainly characterize how frequently a node meets some other node. Thus, our DTN model provides us a fundamental insight in that we can represent an arbitrary delay tolerant network with *node speed* and *radio* characteristics using a single parameter capturing the pairwise inter-contact rate λ.

We then move on to consider a class of random mobility patterns, known to cause a two-phase inter-contact distribution [15, 2]. Cai and Eun [2] found that the shorter the average flight distance, the stronger the correlations in the mobility pattern, leading to a *heavier* power-law head in random mobility (e.g., as in Brownian or random walk mobility models). For an intuitive appreciation of this behavior, consider sightseers around Time Square in Manhattan. A random sightseer tends to encounter other sightseers, because they have the same interests, and their movements are correlated. This correlation disappears when they are no longer in the same "common interest" domain. For example, after Time Square sightseeing, they take off to other attractions. As tourists are taking these long flights, they may still meet each other like "ships that pass in the night." In the Time Square phase, the motions are correlated and the inter-encounter time distributions tend be heavy tailed (due to sightseeing locality). The "ship in the night" encounters, on the other hand, can be modeled as Poisson, with exponential inter-encounter distributions.[2]

Note that in the tourist case, the correlation is caused by a common interest for a local attraction. In more general cases (e.g., vehicles) the local correlation may be caused by "interest" in a common resource, e.g., the road. During a rush hour, vehicles drive in columns on multiple lanes for a while, passing each other in turns, with a strong motion correlation imposed by road constraints, until at the next intersection they depart in different directions, to join other correlation domains. Karagiannis et al. [15] observed such a correlation behavior in their vehicular contact traces, leading to power law distributions. In view of these recent discoveries, one of the main objectives of this work is to study the impact of the two-phase distribution on DTN performance.

Using this extended DTN model, we make the following contributions. First, we report a generalized capacity and delay scaling law of the two-hop routing algorithm based on the inter-contact rate parameter, and the degree of motion correlation to provide a new insight to the fundamental properties of DTNs. Second, we analyze the impact of finite buffer on the capacity/delay scaling properties of DTNs. Finally, we validate our analytic results with a simulation study.

The following is the preview of the key results under a DTN scenario that has n nodes in a unit square area with radio range $r = O(1/\sqrt{n})$, the pairwise inter-contact rate of λ, and the average flight distance of $\Omega(r)$ where nodes travel in the order of its radio range $r = O(1/\sqrt{n})$, which is a reasonable assumption; e.g., a tourist in Manhattan may travel several blocks for each movement (comparable to WiFi radio range).

- The per-node throughput of a two-hop relay routing protocol can be represented as the product of the aggregate meeting rate $(n\lambda)$ and the contact duration $(D_c = \Theta(r/v))$; thus, it scales as $\Theta(n\lambda D_c) = \Theta(nr^2)$. Grossglauser and Tse's result is a special case when the radio range is $\Theta(1/\sqrt{n})$, thus achieving $\Theta(1)$ throughput. Our results show that the node speed and the degree of correlation in mobility patterns do not affect the achievable throughput.

- The average delay of two-hop relay routing is in the range of $[\Theta(\frac{1}{\lambda}), \Theta(\frac{\log n}{\lambda})]$ depending on the degree of motion correlation. Here, the lower bound is for the case when the nodes meet with an exponential inter-contact time. The upper bound is of the case of a random walk with flight length of $\Theta(r)$. As the degree of motion correlation decreases (i.e., flight distance increases), the average delay decreases monotonically. Our results show that the delay increase due to motion correlations can be bounded by the factor of $\Theta(\log n)$ compared to the delay in the exponential case.

- We show that the two-hop relay routing requires the buffer space ranging in $[\Theta(\frac{nr}{v}), \Theta(\frac{n \log n}{rv})]$; the stronger the correlations of the mobility patterns, the higher the amount of buffer space. Also, we show that such correlations cause the burstiness in the inbound/outbound

[2]Rhee et al. [25] showed that the human walks resemble a "truncated" form of Levy walks (or a truncated power-law). Through their Levy walk mobility model with properly chosen parameters, they could observe the two-phase inter-contact distribution. Chaintreau et al. [3] also showed the two-phase distribution from their human mobility traces. Besides, Conan et al. [4] showed that several mobility traces contain significant fraction of contact pairs following exponential distributions.

relay traffic. Under a simple policy of admitting a packet whenever there is available space, we show that the per-node throughput with constrained buffer size of K in the network is in the range of $[\Theta(\frac{rvK}{\log n}), \Theta(rvK)]$ depending on the degree of motion correlation. This is a tighter bound than the previous results by Herdtner et al. [13].

The rest of the paper is organized as follows. In Section 2, we present the network model. In Section 3, we show the analysis of the DTN routing protocols. We also investigate various DTN design parameters and their impacts on the scaling properties. In Section 4, we validate our results via simulations. Finally, we present the conclusion in Section 5.

2. NETWORK MODEL

In this section, we review the system model used for analysis. We first present the communication model; then provide a simple mobility model with which we represent a DTN in general; and finally we describe our DTN model.

2.1 Communication Model

We use the protocol model to abstract the interference between transmissions [12]. Suppose that node i transmits to node j. Node j receives the transmission successfully if every other node that transmits simultaneously is at a distance at least $(1 + \Delta)r(n)$ from j where Δ is some positive number and $r(n)$ is the radio range. Each node $i \in \{1, 2, \cdots, n\}$ is randomly assigned a destination node $d_i \neq i$, so there are a total of n source-destination pairs.

For a given scheduling algorithm π, a throughput $\gamma > 0$ is said to be feasible (or achievable) if every node can send at a rate of γ bits per second to its chosen destination. Let $T^\pi(n)$ denote the maximum feasible per-node throughput under scheduling algorithm π. The delay of a packet in a network is the time for a packet to reach the destination after it leaves the source. Let $D^\pi(n)$ denote the average packet delay for a network with n nodes under scheduling algorithm π. Note that a scheduling algorithm is *stable* if the rate $T^\pi(n)$ is satisfied by all users such that one's queue does not grow infinity, i.e., $D^\pi(n)$ is bounded.

2.2 Mobility Model

We consider two mobile nodes A and B, each of which moves according to some mobility model in a unit square area of size 1×1 denoted as Ω. Let $A(t), B(t) \in \Omega$ be the position of the node A and B at time t, respectively. The pairwise inter-contact time, which is the time interval between two successive encounters of a pair of nodes, can be formally described as follows [3, 2].

DEFINITION 1. *The inter-contact time T_I of a node A with a node B is defined by*

$$T_I = \inf_{t>0}\{t : \|A(t) - B(t)\| \leq r\}, \qquad (1)$$

given that

$$\|A(0) - B(0)\| = r \text{ and } \|A(0^+) - B(0^+)\| > r \qquad (2)$$

where $\| \cdot \|$ is the Euclidian norm in R^2.

If we remove the given condition (2), we measure the time until next encounter to node B from a randomly chosen position at $t = 0$. We define this as the first passage time T_R of node A to node B; to be precise, it is the residual life

time of the inter-meeting time [3, 2]. We recognize that the larger the inter-contact interval, the higher the probability that a random sample observes that interval, which is also known as an inspection paradox or a length bias. Recall that the inter-contact time has a finite mean because of our exponential "tail" assumption. In [16], the distribution of the residual life time is given as

$$Pr(T_R > t) = \frac{1}{E[T_I]} \int_t^\infty P[T_I > s]ds \qquad (3)$$

and the mean residual life time is given as

$$E[T_R] = \frac{E[T_I^2]}{2E[T_I]} \qquad (4)$$

Thus, the mean first passage time is mainly a function of the first and second moments of the inter-contact time distribution.

In our analysis, we consider a class of random mobility models where each node independently makes its own decision; e.g., random direction mobility where a mobile node selects a uniform random direction from $[0, 2\pi]$ and a speed v and moves until it hits the boundary of a domain. Groenevelt et al. [10, 9] showed that the inter-contact stochastic process of these mobility models can be captured using an independent homogeneous Poisson process with meeting rate λ. We present the Theorem 4.2.1 from [9] to provide a basis for estimating the λ value for different mobility models.

THEOREM 1. *Given that two nodes move randomly in a 1×1 unit square with the average speed v, if the transmission range $r \ll 1$ and the position of a node at time $t + \Delta$ is independent of its position at time t for small Δ, then the inter-contact time between two nodes is exponentially distributed with parameter $\lambda = \alpha rv$ where α is a constant.*

Researchers have recently found that there is the critical timescale in the inter-contact distribution at which the transition from power-law to exponential takes place (or power-law distribution with exponential cut-off) [15, 2]. Cai and Eun [2] used Isotropic Random Walk (IRW), a variant of random direction mobility where a node chooses a *random step-length L*, a random angle, and random speed. By taking an appropriate step-length distribution, IRW can approximate different mobility patterns: random direction mobility with large L and Brownian motion mobility with small L (over small time intervals). Given the fact that the smaller the random step-length, the stronger the correlations in mobility patterns, they proved that stronger correlations lead to a heavier power-law head in the inter-contact time distribution. Also, they found that there exists an *invariance property* of several contact-based metrics such as inter-contact and inter-any-contact time in that the *averages of those metrics do not depend on the degree of correlation in the mobility patterns* [2]. Thus, our model of using the average meeting rate λ is still valid. They also showed that the stronger the correlation, the higher the value of n-th order moment for $n > 1$ (see Theorem 2), and we discuss the implication of power-law heads in the later sections.

THEOREM 2. *Let X and Y be random variables representing mobility statistics. If Y has stronger correlation than X, and $E[X] = E[Y]$, then $E[\varphi(X)] \leq E[\varphi(Y)]$ for all convex function φ.*

REMARK 1. *It is known that the stationary distribution of node positions for all the above mobility models is a uni-*

form distribution [23]. *This allows us to model a snapshot of a mobile network (or a spatial configuration of nodes in the network) using a homogeneous Poisson point process with intensity n where n is the number of nodes in the network. We denote the process as $\{N(A)\}_{A \in 1 \times 1}$ and it satisfies the following conditions: (1) for every $A \in 1 \times 1$, $N(A) \sim Poisson(n|A|)$ such that we have:*

$$Pr(N(A) = k) = \frac{e^{-n|A|}(n|A|)^k}{k!} \qquad (5)$$

and (2) for every finite collection $\{A_1, A_2, \cdots, A_n\}$ of disjoint subsets of a unit square area, $N(A_1), N(A_2), \cdots, N(A_3)$ are independent of one another.

2.3 DTN Model

We model an arbitrary DTN in a unit area of (1×1) using the pairwise inter-contact rate $\lambda = \Theta(rv)$ where r is radio range and v is speed. We can map an arbitrary delay tolerant network to a unit area by relatively scaling the radio range and speed. Also, we note that Theorem 1 shows that the pair-wise inter-contact rate is independent of the number of nodes.

The relationship between radio range and speed is important, as it determines the contact duration. The asymptotic contact duration $D_c(n)$ can be easily derived from the results in [27, 30] as $D_c(n) = \Theta(r(n)/v(n))$. We note that the contact duration determines the amount of data two nodes can exchange. Thus, it places an upper bound on the size of a packet. In the literature [5, 6, 28], it is typically assumed that the speed of a node is in the same order as the radio range such that the contact duration of two nodes is constant (and so is the packet size).[3] For instance, with radio range $r = 1/\sqrt{n}$, we set the speed $v = 1/\sqrt{n}$ (i.e., $\lambda = 1/n$). In fact, other cases are still possible. When the speed is asymptotically smaller than the radio range, we can still maintain the constant packet size, yet the number of packets that one can transmit for a given contact scales as $\omega(1)$. When the speed is asymptotically greater than the radio range, the packet size must be scaled down such that a packet can be transferred for a given contact duration.[4]

The radio range determines the number of simultaneous transmissions, or the network-wide aggregate throughput. Since it is approximately the same as the total number of non-overlapping circles with radius r that fills 1×1 area, the network-wide aggregate throughput $\mathcal{T}$ is bounded by $\Theta(1/r^2)$. The per-node throughput bound is simply given by dividing the aggregate throughput by the total number of communication pairs. Therefore, the aggregate throughput can be expressed as $\mathcal{T} \leq \Theta(1/r^2)$. For a DTN with the radio range r, the upper bound of the per-node throughput can be maximized, when the number of nodes is in the same order as the aggregate throughput, i.e., $\Theta(1/r^2) = \Theta(n)$ and thus, $r = \Theta(1/\sqrt{n})$. In this paper, we analyze more general scaling behavior with the radio range of $O(1/\sqrt{n})$.

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[3]Note that Ying et al. [31] showed that there are different time-scale of mobility, namely fast and slow mobility. For slow mobiles, node mobility is assumed to be much slower than data transmissions and thus, multi-hop transmissions are feasible in single time slot (the packet size can be scaled down as n increases). In contrast, we use the contact duration to measure how fast the speed is. Since we do not use multi-hop routing, our scenario belongs to fast mobility in their framework.

[4]However, this may not be feasible due to Doppler shifts; i.e., channel coherence time may be shorter than the packet transmission time, and thus, a node may suffer from fast fading.
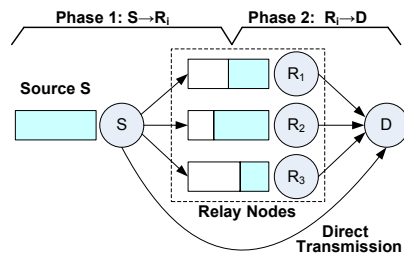

Figure 1: An illustration of the two-hop relay algorithm.

In the remainder of this paper, we slightly abuse the asymptotic notation as follows to represent the scaling properties in the DTN region. For instance, when we say that the per-node throughput of the two-hop relay scheme is $\Theta(nr^2)$, this statement is true when r scales with n. However, when λ is fixed (i.e., both radio range and node speed are fixed), it is true when $n \leq 1/r^2$. This conditional rule applies to all the asymptotic notations in this paper.

3. DTN ROUTING ANALYSIS

We first present the capacity and delay scaling laws of the two-hop relay routing model using our DTN model. This result is generalization of the previous results by Grossglauser and Tse [11]. We then discuss the impact of finite buffer.

3.1 Capacity Analysis

We briefly present the two-hop relay algorithm proposed by Grossglauser and Tse [11] for completeness. Whenever two nodes encounter one another (i.e., within the radio range), we do the following. If they are a source-destination pair, the source transmits a packet to the destination (*direct transmission*). Otherwise, either a source node sends a new packet to a relay node (*Phase 1: Relay*), or a relay node delivers a packet to the destination (*Phase 2: Delivery*). The overall procedure is illustrated in Figure 1. In this scheme, a relay node has a separate queue for each source and destination pair, and it may store multiple packets in the queue.

LEMMA 1. *For a DTN with the exponential inter-contact rate $\lambda = O(1/n)$, the per-node capacity of the two-hop relay scheme is $\Theta(n\lambda D_c) = \Theta(nr^2)$.*

PROOF. Consider a pair of nodes: source i and destination d_i. During a small time interval Δt, a random node j encounters the destination with the probability $\lambda \Delta t + o(\Delta t)$. In the network setting that we consider, there could be at most a constant number of nodes (denoted as $\xi = O(1)$) within one's wireless contention domain with high probability. Assuming that the chance of transmission is equally shared by ξ interfering nodes under the protocol model [17, 6, 5, 13], node j can successfully deliver a packet with the probability $\lambda \Delta t/\xi$. Here, we are interested in the event that the destination d_i is scheduled to receive node i's packet at time t. Let an indicator random variable $M_i(\Delta t, n)$ denote this event. Since d_i can meet any of the relay nodes, we have:

$$Pr\{M_i(\Delta t, n) = 1\} \qquad (6)$$

$$= \sum_{j=1, j \neq d_i}^{n} Pr\{\text{node } j \text{ delivers a packet during } \Delta t\} \qquad (7)$$

$$\approx \frac{(n-1)\lambda \Delta t}{\xi} \qquad (8)$$

For each contact, a node can transfer packets for the contact duration of $D_c = \Theta(r/v)$, and thus, the per-node throughput is given as

$$T(n) = \frac{\mathbb{E}[M_i(\Delta t, n)]}{\Delta t} D_c = \frac{(n-1)\lambda \Delta t}{\xi} \frac{1}{\Delta t} D_c \quad (9)$$

$$= \Theta(n\lambda D_c) = \Theta\left(nrv\frac{r}{v}\right) = \Theta(nr^2) \quad (10)$$

$\square$

The result shows that the throughput is simply the product of the aggregate meeting rate $(n\lambda)$ and the contact duration (D_c); i.e., it is determined by how often a destination node encounters a random relay node, and how long it can transfer packets to the node. Grossglauser and Tse's results of $\Theta(1)$ can be achieved when the radio range is $\Theta(1/\sqrt{n})$. Hereafter, we use this aggregate meeting rate to find the throughput bounds. Note that this simple relationship enables us to understand the maximum throughput of an arbitrary DTN; for a given radio range, the throughput linearly increases with the number of nodes. For instance, Zhao et al. [32] introduced additional relay nodes called throwboxes to increase the throughput of a DTN.

THEOREM 3. *The two-phase mobility pattern (consisting of an exponential head and a power-law tail) does not change the capacity of the two-hop relay scheme.*

PROOF. Since there is a motion correlation, a destination node will meet a certain relay node more often for a given period of time, and then lose sight of it for a long-time interval. Nevertheless, each source's traffic is spread uniformly among all other nodes, because nodes have uniformly distributed motion patterns over the area. In fact, the degree of motion correlation determines the burstiness of inbound/outbound traffic into the destination and out of the source. The stronger the correlations, the higher the chances for a source to meet candidate relay nodes in a short period, thus increasing the burstiness of the outbound traffic. The same applies to destinations meeting relay nodes carrying packets for them. Although the destination encounter frequency with relay nodes follows a bimodal distribution, the asymptotic capacity results do not change; namely, as long as the long term average meeting rate (of meeting any relay nodes) is the same. To see this, consider a spatial Poisson point process where Poisson intensity is given as $nA = n\pi r^2$. For a given network snapshot, a destination node encounters a relay node with probability $Pr(N(A) \geq 1) = n|A| + o(|A|) = n\pi r^2 = \Theta(nr^2)$. Since every time slot can be utilized with this probability, the per-node throughput is simply $\Theta(nr^2)$. Thus, we conclude that this encounter process with uniform spread of traffic warrants that a node can achieve the per-node throughput of $\Theta(nr^2)$. $\square$

3.2 Delay Analysis

We now look at the delay of the two-hop relay scheme. For now, we assume that a communication pair is given an infinite size buffer. Consider a pair of nodes: source i and destination d_i. The relay delay is the expected time for a packet at the source to be delivered to the destination d_i. If the source encounters the destination first, it can be delivered directly. In our model, this happens with probability $1/n$, and the average delay is denoted as D_d. Otherwise, the packet will be delivered via a relay node with probability $1 - 1/n$. In this case, the average delay is composed of the average source-to-relay delay (D_{sr}) and the average relay-to-destination delay (D_{rd}). Thus, the expected delay can be expressed as follows:

$$D(n) = \frac{1}{n}D_{sd} + \frac{n-1}{n}\left(D_{sr} + D_{rd}\right) \quad (11)$$

We consider two extreme cases to better understand the average delay of the two-hop relay scheme: as the best case of two-phase mobility, we look at a random direction model that shows exponential inter-contact time (i.e., flight length of $O(1)$); and as the worst case of two-phase mobility, we look at a random walk on $1/r \times 1/r$ torus where the step length is $r < 1$. Note that for a node speed of v, each step takes r/v.

Since the average inter-contact time is the same regardless of motion correlation by the invariance property, D_{sd} and D_{sr} in Equation 11 can be easily found using an exponential inter-contact model. A source node encounters a destination node with rate λ, and any potential relay nodes with rate $(n-2)\lambda$ (i.e., minimum of $n-2$ exponential random variables). Thus, the average source-to-destination delay and the average source-to-any-relay-node delay are given as $D_{sd} = \Theta(\frac{1}{\lambda})$ and $D_{sr} = \Theta(\frac{1}{n\lambda})$, respectively.

However, we notice that the delay from a relay node to the destination (D_{rd}) is a function of motion correlation. When a source node delivers a packet to the relay node, it samples a random point of the inter-contact instances between a relay node and a destination node. As shown in Equation 4, the time for a relay node to deliver a packet to a destination node is defined as the residual inter-contact time, or a first passage time; i.e., due to the length bias, when a source node sends a packet to a relay, it is more likely to sample a longer inter-contact interval between a relay node and a destination node.

Having said that the following theorem summarizes the average delay scaling with correlated motion patterns.

THEOREM 4. *The average delay with an arbitrary flight distance of $\Omega(r)$ is in $[\Theta(\frac{1}{\lambda}), \Theta(\frac{\log n}{\lambda})]$. Here, the lower bound is for the case when the nodes meet with an exponential inter-contact time; and the upper bound is of the case of a random walk with flight length of $\Theta(r)$. As the degree of motion correlation decreases (i.e., flight distance increases), the average delay decreases monotonically.*

PROOF. As shown in Equation 4, for an arbitrary inter-contact distribution with finite mean, the mean residual inter-contact time is given as $\frac{E[T_I^2]}{2E[T_I]}$. The second moment of the inter-contact time for exponential cases is simply $\Theta(1/\lambda^2)$. The mean residual inter-contact time is $D_{rd} = \frac{E[T_I^2]}{2E[T_I]} = \frac{1/\lambda^2}{2/\lambda} = \Theta(1/\lambda)$. For a random walk, El Gamal et al. [7] showed that the second moment of the inter-contact time takes $\Theta(\frac{-\log r}{r^4})$ steps. By the invariance property, the average inter-contact time is $E[T_I] = \Theta(1/\lambda) = \Theta(\frac{1}{rv}) = \Theta(1/r^2)$ steps. Recall that each step takes $\Theta(r/v)$. The mean residual inter-contact time of a random walk is $\frac{E[T_I^2]}{2E[T_I]} = \frac{-\log r}{2r^4}r^2 = \frac{-\log r}{2r^2}$ steps. As each step takes $\Theta(\frac{r}{v})$, the mean residual inter-contact time is given as $D_{rd} = \Theta(\frac{-\log r}{r^2}\frac{r}{v}) = \Theta(\frac{-\log r}{\lambda})$. Compared to the exponential case, a random walk results in the mean residual inter-contact time that is greater by the factor of $\Theta(-\log r)$. As we have $r = O(1/\sqrt{n})$, the lower/upper bound can be represented as $D_{rd} = \Theta(\frac{1}{\lambda})$ and $D_{rd} = \Theta(\frac{\log n}{\lambda})$, respectively. By plug-

ging D_{rd} into Equation 11, we find that the average delay $D(n)$ scales as D_{rd}.

We can also consider the queueing delay using the techniques used in [7]. The overall system can be modeled using a GI/GI/1-FCFS queue. Both arrival and departure processes can be modeled using the inter-contact time distribution. Let X and Y denote the arrival and departure processes respectively. By Kingman's upper bound [26] on the average delay for a GI/GI/1-FCFS queue, the average delay is upper bounded as $O(\frac{E[X^2]+E[Y^2]}{E[X]})$. From this, we find that for the exponential case, the average delay is upper bounded by $O(1/\lambda)$ and for the random walk case, it is upper bounded by $O(\frac{-\log r}{r^4}r^2) = O(\frac{-\log r}{r^2})$ steps $= O(\frac{\log n}{\lambda})$. In the above, we show that the average delay for the exponential case and the random walk case respectively is given as $\Theta(\frac{1}{\lambda})$ and $\Theta(\frac{\log n}{\lambda})$ even without considering queueing delay. Hence, we conclude that the average delay for the exponential case and the random walk case is tightly bounded as $\Theta(\frac{1}{\lambda})$ and $\Theta(\frac{\log n}{\lambda})$ respectively.

We now show that the average delay monotonically decreases as the degree of motion correlations decreases. Let X and Y denote random variables representing inter-contact time of mobility patterns with the average step length L_X and L_Y, respectively and $L_X \geq L_Y$. This means that the mobility pattern represented by Y has stronger correlation than that represented X. For an arbitrary convex function φ, Theorem 2 shows $E[\varphi(X)] \leq E[\varphi(Y)]$. The mean residual time in Equation 4 is a convex function of the inter-contact time (i.e., the second moment), thus satisfying the inequality. This proves the monotonicity of the average delay with the degree of motion correlation. $\square$

3.3 Buffer Requirements

For a given node pair, we can easily find the average packet queue in the network using the Little's law: the product of the per-node throughput and the average packet life time. Since the packet life time is equal to the average delay, the average number of packets for a given pair (i.e., the number of buffers required to support a given pair flow) is determined by the throughput-delay product in a DTN. From this observation, the average buffer space required for two-hop relay with an arbitrary degree of motion correlation ranges in between $\Theta(nr^2) \times \Theta(1/\lambda) = \Theta(\frac{nr}{v})$ and $\Theta(nr^2) \times \Theta(\log n/\lambda) = \Theta(\frac{nr \log n}{v})$.

Given finite buffer space of size K per source in the network, we now want to find the throughput bound. We assume that a buffer replacement algorithm is *not* used. Instead, we use a simply packet admission policy in which a relay node accepts a relay packet from a source node if there is an available space in the relay node. Since the stationary distribution of nodes is uniform, and by the invariance property, the inter-contact time does not change with the degree of motion correlation, each node has equal opportunities of encountering other nodes on average. Thus, for a given set of free buffer space, source nodes will equally share the space. Given this, the following theorem shows the throughput per source with finite buffer.

THEOREM 5. *Given finite buffer space of size K per source in the network, the throughput per source is in range of $[\Theta(\frac{rvK}{\log n}), \Theta(rvK)]$.*

PROOF. The size of buffer space per source, N_B to sustain the maximum throughput is given as $[\Theta(\frac{nr}{v}), \Theta(\frac{nr \log n}{v})]$. For a given relay node, the limited buffer space of K is equally shared by n sources. Since we assume that each source has an equal opportunity of utilizing the buffer space, a relay node can receive a packet to a random destination with the expected probability of K/N_B. Using the same proof technique as in Theorem 1, we find that the throughput per source ranges in $[\Theta(\frac{rvK}{\log n}), \Theta(rvK)]$. Given finite buffer space, the results show that the mobility with stronger correlations will result in lower throughput. $\square$

4. SIMULATIONS

We validate our analytic results by packet-based simulations using QualNet $v3.9.5$. We measure the inter-contact time, the average contact duration, and the throughput of two-hop relay routing. We show the impact of motion correlation using the inter-any-contact time and the buffer utilization.

4.1 Simulation Setup

We use the Isotropic Random Walk (IRW) where each node chooses a *random step-length* and a random angle, and moves to a chosen direction at the constant speeds of 20m/s or 30m/s. Random step-length follows an exponential distribution with the mean of Lm. Nodes are moving in an area of size 5000m × 5000m. We use 802.11b with the two-ray ground path-loss propagation model, 250m transmission range, and 2Mbps transmission rate. We vary the number of nodes from 10 to 100 by 10 node increments. We implement the two-hop relay routing protocol, in which a node maintains a separate queue for each destination. When a node encounters another node, if the encountered node is a destination, the node will keep sending packets until the link breaks; otherwise, the node makes a packet by packet forwarding decision; i.e., with the same probability, the node either relays its own packet to a relay node or it delivers a relay packet to the encountered destination. To measure the maximum throughput, we randomly choose a single pair of nodes and generate packets using the Constant Bit Rate (CBR) traffic in QualNet. We warm up each simulation run for 10,000s to remove the effect of the initial startup phase. Unless otherwise mentioned, reported results are the averages of 50 runs with different random seeds and are presented with the 95% confidence interval. The duration of each run is 100,000s.

4.2 Simulation Results

We measure the pairwise inter-contact time and plot the Complementary Cumulative Distribution Functions (CCDF) in Figure 2 and Figure 3. Note that the results are presented in the log-log scale, and small figures are presented in the log-linear scale. The figures show that there is a power-law head when we have short step length of $L = 250m$. As the degree of correlation in mobility patterns disappears (i.e., as the average step length L increases), the inter-contact time distribution becomes an exponential distribution (e.g., $L = 1000m$). We also report the average and standard deviation of inter-contact time. When the speed is 20m/s, the mean inter-contact time is given as 1950.18s (dev 3294.63s) for $L = 250m$ and 1953.77s (dev 2277.63s) for $L = 1000m$. When the speed is 30m/s, it is given as 1365.66s (dev 2257.82s) for $L = 250m$ and 1339.39s (dev 1546.508s) for $L = 1000m$. Our results show that the average value is about the same, but as shown earlier, the variance of inter-contact time increases, as the degree of motion correlation increases. Also, we report that a node with the maximum
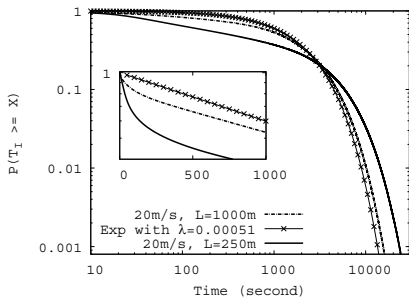
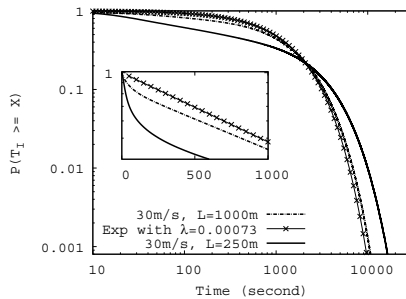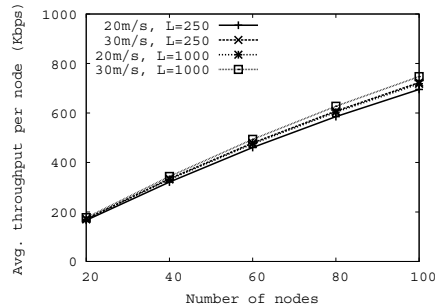**Figure 2: Inter-contact time CCDF (20m/s, log-log)** **Figure 3: Inter-contact time CCDF (30m/s, log-log)** **Figure 4: Average per-node throughput**
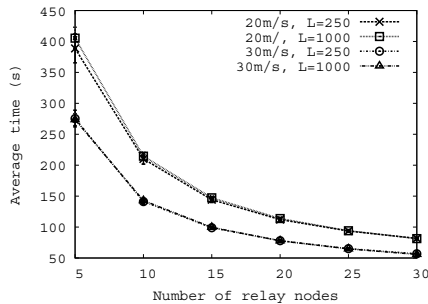

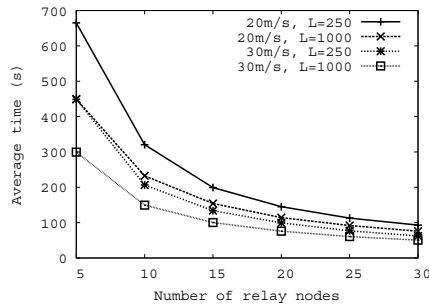**Figure 5: Average inter-any-contact time** **Figure 6: Average residual inter-any-contact time**

speed of 20m/s and 30m/s has the average contact duration of about 14.70s and 10.01s, respectively.

We measure the per-node throughput by increasing the CBR traffic rate (i.e., packets/sec) of a single source-destination pair. In Figure 4, we present the measured throughput as a function of the number of relay nodes. Our results validate the analytic results in Theorem 1: (1) when the network parameters are given (i.e., λ is fixed), the per-node throughput linearly increases as the number of relay nodes increases ($n\lambda D_c$); and (2) the throughput is not affected by the node speed or the motion correlation.

To show the impact of motion correlation, we present the average inter-any-contact time (Figure 5) and the average residual inter-any-contact time (Figure 6). For a given node, the inter-any-contact time measures the time intervals for a node to encounter any of the k nodes. Given this, the first and second moments of the measured inter-any-contact time are then used to calculate the average residual inter-any-contact time using Equation 4. Figure 5 shows that the average inter-any-contact time decreases as the number of node increases, and it is not influenced by the degree of motion correlation. In Figure 6, however, we notice that the stronger the correlations in mobility patterns (e.g., L=250m), the longer the delay "tail" and the larger the residual inter-any contact time.

Finally, we analyze the impact of motion correlation on the buffer utilization. We can check the buffer utilization by measuring the number of packets in a relay node's buffer during a simulation. We present the buffer utilization of a randomly selected node in Figure 7. We notice that the case with $L = 250m$ has larger variance in buffer utilization. Given a small flight average distance, nodes are "trapped" in a random region, and thus, nodes in that region will meet more often before they depart. In that time interval, packets remain in the neighborhood set, and buffer occupancy increases. For the case with $L = 1000m$, there is less cor-

relation, and buffer occupancy is practically uniform over the entire simulation time. To validate this observation, we measure the number of consecutive encounters to a source node by a relay node before it encounters a destination node, or vice versa. Figure 8 shows the cumulative distribution of the number of consecutive encounters. The figures show that the larger the average flight distance, the smaller the number of consecutive encounters. For instance, the average number of consecutive encounters for 30m/s is 3.2 and 2.3 for $L = 250m$ and $L = 1000m$ respectively. Our results confirm that such correlations in mobility patterns indeed cause the burstiness in inbound/outbound traffic at a relay node.

5. CONCLUSION

We studied the capacity/delay scaling properties of DTN routing protocols in mobile ad hoc networks with intermittent connectivity. We represent a DTN under a class of random mobility models such as random direction, using a pairwise inter-contact time and the degree of motion correlation. Using this unified framework, we generalized the scaling behavior of two-hop relay routing and studied the impact of finite buffer on the scaling behavior. Our results show that in the scenarios under consideration: (1) the per-node throughput is not affected by the node speed and the degree of motion correlation; (2) the motion correlation increases variance in the inter-contact time, and consequently, it increases the average delay, buffer requirements, and the burstiness of inbound/outbound traffic at the relay nodes; and (3) buffer requirements can be represented as the throughput-delay product, which allows us to analyze the trade-offs between buffer and throughput.
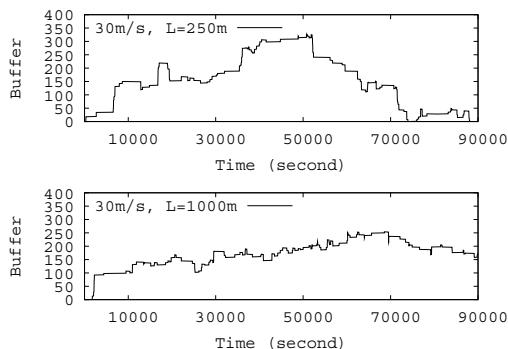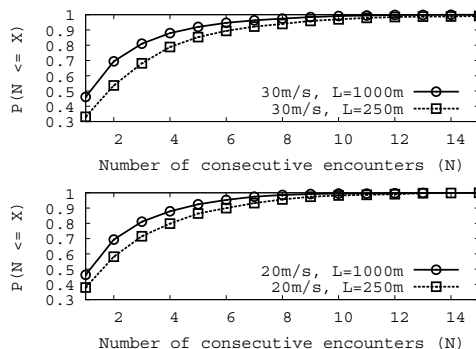
Figure 7: Buffer utilization over time

Figure 8: Cumulative distribution of the number of consecutive encounters

6. REFERENCES

[1] J. Burgess, B. Gallagher, D. Jensen, and B. N. Levine. MaxProp: Routing for Vehicle-Based Disruption-Tolerant Networks. In *INFOCOM'06*, Barcelona, Spain, Apr. 2006.

[2] H. Cai and D. Y. Eun. Toward Stochastic Anatomy of Inter-meeting Time Distribution under General Mobility Models. In *MobiHoc'08*, Hong Kong, May 2008.

[3] A. Chaintreau, P. Hui, J. Crowcroft, C. Diot, R. Gass, and J. Scott. Impact of Human Mobility on the Design of Opportunistic Forwarding Algorithm. In *INFOCOM'06*, Barcelona, Spain, Apr. 2006.

[4] V. Conan, J. Leguay, and T. Friedman. Characterizing Pairwise Inter-contact Patterns in Delay Tolerant Networks. In *ACM Autonomics'07*, Rome, Italy, Oct. 2007.

[5] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves. Throughput-Delay Analysis of Mobile Ad-hoc Networks with a Multi-Copy Relaying Strategy. In *SECON'04*, Santa Clara, CA, Oct. 2004.

[6] A. E. Gamal, J. Mammen, B. Prabhakar, and D. Shah. Throughput-Delay Trade-Off in Wireless Networks. In *INFOCOM'04*, Hong-Kong, Mar. 2004.

[7] A. E. Gamal, J. Mammen, B. Prabhakar, and D. Shah. Optimal Throughput – Delay Scaling in Wireless Networks – Part I: The Fluid Model. *IEEE Transactions on Information Theory*, 52(6), June 2006.

[8] M. Garetto, P. Giaccone, and E. Leonardi. Capacity Scaling in Delay Tolerant Networks with Heterogeneous Mobile Nodes. In *MobiHoc'07*, Quebec, Canada, Sept. 2007.

[9] R. Groenevelt. Stochastic Models in Mobile Ad Hoc Networks. Technical report, University of Nice, Sophia Antipolis, INRIA, 2006.

[10] R. Groenevelt, P. Nain, and G. Koole. The Message Delay in Mobile Ad Hoc Networks. In *Performance'05*, Juan-les-Pins, France, Oct. 2005.

[11] M. Grossglauser and D. N. C. Tse. Mobility Increases the Capacity of Ad Hoc Wireless Networks. *IEEE Transactions on Networking*, 10(4), Aug. 2002.

[12] P. Gupta and P. R. Kumar. The Capacity of Wireless Networks. *IEEE Transactions on Information Theory*, 46(2), 2000.

[13] J. D. Herdtner and E. K. P. Chong. Throughput-Storage Tradeoff in Ad Hoc Networks. In *INFOCOM'05*, Miami, FL, Mar. 2005.

[14] S. Jung, U. Lee, A. Chang, D. Cho, and M. Gerla. BlueTorrent: Cooperative Content Sharing for Bluetooth Users. In *PerCom'07*, White Plains, NY, Mar. 2007.

[15] T. Karagiannis, J.-Y. L. Boudec, and M. Vojnovic. Power Law and Exponential Decay of Inter Contact Times Between Mobile Devices. In *MobiCom'07*, Montréal, QC, Canada, Sept. 2007.

[16] L. Kleinrock. *Queueing Systems: Theory, Volume 1.* Wiley-Interscience, 1975.

[17] S. R. Kulkarni and P. Viswanath. A Deterministic Approach to Throughput Scaling in Wireless Networks. *IEEE Transactions on Information Theory*, 50(6):1041–1049, 2004.

[18] U. Lee, S. Y. Oh, K.-W. Lee, and M. Gerla. RelayCast: Scalable Multicast Routing in Delay Tolerant Networks. In *ICNP'08*, Orlando, Florida, Oct. 2008.

[19] U. Lee, J.-S. Park, E. Amir, and M. Gerla. FleaNet: A Virtual Market Place on Vehicular Networks. In *V2VCOM'06*, San Jose, CA, July 2006.

[20] U. Lee, J.-S. Park, J. Yeh, G. Pau, and M. Gerla. CodeTorrent: Content Distribution using Network Coding in VANETs. In *MobiShare'06*, Los Angeles, CA, Sep. 2006.

[21] X. Lin, G. Sharma, R. R. Mazumdar, and N. B. Shroff. Degenerate Delay-Capacity Trade-offs in Ad Hoc Networks with Brownian Mobility. *IEEE/ACM Transactions on Networking (TON)*, 14(SI):2777–2784, 2006.

[22] X. Lin and N. B. Shroff. The Fundamental Capacity-Delay Tradeoff in Large Mobile Wireless Networks. In *MedHocNet'04*, Bodrum, Turkey, Jun. 2004.

[23] P. Nain, D. Towsley, B. Liu, and Z. Liu. Properties of Random Direction Models. In *INFOCOM'05*, Miami, FL, Mar. 2005.

[24] M. J. Neely and E. Modiano. Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks. *IEEE Transactions on Information Theory*, 46(2), June 2005.

[25] I. Rhee, M. Shin, S. Hong, K. Lee, and S. Chong. On the Levy-walk Nature of Human Mobility. In *INFOCOM'08*, Phoenix, AZ, Apr. 2008.

[26] R. W. Rolff. *Stochastic Modeling and the Theory of Queues.* Prentice Hall, 1989.

[27] P. Samar and S. B. Wicker. On the Behavior of Communication Links of a Node in a Multi-Hop Mobile Environment. In *MobiHoc'04*, Tokyo, Japan, May 2004.

[28] G. Sharma, R. Mazumdar, and N. Shroff. Delay and Capacity Trade-offs in Mobile Ad Hoc Networks: A Global Perspective. In *INFOCOM'06*, Barcelona, Spain, Apr. 2006.

[29] G. Sharma and R. R. Mazumdar. Delay and Capacity Trade-off in Wireless Ad Hoc Networks with Random Way-point Mobility. Technical report, Purdue University, 2005.

[30] S. Xu, K. Blackmore, and H. Jones. Mobility Assessment for MANETs Requiring Persistent Links. In *WitMeMo'05*, Seattle, WA, Jun. 2005.

[31] L. Ying, S. Yang, and R. Srikant. Coding Achieves the Optimal Delay-Throughput Tradeoff in Mobile Ad Hoc Networks: Two-Dimensional I.I.D. Mobility Model with Fast Mobiles. In *WiOpt'07*, Limassol, Cyprus, Apr. 2007.

[32] W. Zhao, Y. Chen, M. Ammar, M. Corner, B. Levine, and E. Zegura. Capacity Enhancement using Throwboxes in DTNs. In *MASS'06*, Vancouver, Canada, Oct. 2006.