Analysis of Cell Sojourn Time in Heterogeneous Networks With Small Cells

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***Abstract*—Recently, heterogeneous networks with small cells have been widely used to increase the capacity of mobile systems. In such environments, accurate estimation of the mean cell sojourn time is critical for evaluating the performance of the network and its applications. It is especially important to analyze the cell sojourn times of mobile users as they reside in different network tiers: either macro-cell-only or small-cell-covered areas. But because small cells are deployed in an irregular manner, it is difficult to derive the analytical mean cell sojourn time in a macro-cell-only area. In this letter, we propose a novel approach to resolve this difficulty. We developed a simple but effective trick that approximates the heterogeneous network to a discrete grid so that it becomes tractable, making it possible to derive the analytical mean sojourn time in the macro-cell-only area. Simulation results confirm that the proposed method has excellent accuracy for general random walk mobility models including random waypoint, Brownian motion, tailored Brownian motion, and truncated Levy walk.**

***Index Terms*—Heterogeneous network, hierarchical cellular mobile network, inter-tier mobile access, user mobility, analytical estimation, cell sojourn time, cell residence time, noncontiguous and irregular cell deployment.**

I. Introduction

THE RAPID increase in mobile data traffic is an emerging issue, and small cells are expected to be an effective solution for improving the spatial density of cellular mobile systems [1], [2]. Major cellular operators such as AT&T consider small cells to be a key part of the advanced network toolsets used to achieve performance goals. One important observation is that small cells are deployed on a *plug-and-play* basis [3]; this implies that mobile users may opportunistically access small cell base stations along their travel routes. In such heterogeneous cellular networks, cell sojourn time, i.e., how long a mobile user stays in a given (macro/small) cell, is an important performance metric for planning network resources such as channel frequencies and time slots, and Quality-of-Service (QoS) analysis [4]. Furthermore, cell sojourn time is also a clue that can be useful in reducing the signaling overhead of mobile services, (e.g., location-based services) [5].

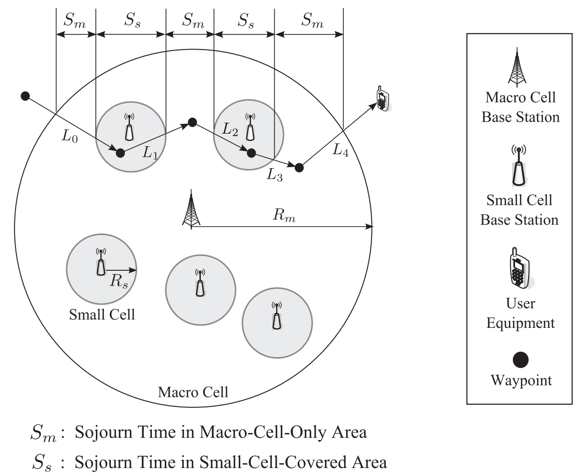

Fig. 1. Example of a two-tier heterogeneous network.

In a heterogeneous network with a mixture of macro and small cells, there are two types of service areas, a macro-cell-only-area (MoA) and a small-cell-covered-area (ScA), as defined in earlier studies [6]–[8]. Fig. 1 illustrates the trajectory of a mobile user, whose cell connectivity alternates between MoA and ScA (i.e., $S_m \rightarrow S_s \rightarrow S_m \rightarrow S_s \rightarrow S_m$) where S_m is the sojourn time in the MoA, and S_s is the sojourn time in the ScA. For this type of network model, it is well known that there are analytically tractable solutions for estimating the mean sojourn time in an ScA because the shape of each small cell is not complicated (e.g., circles) [9]–[11]. However, the shape of an MoA is likely to be irregular because of the arbitrary installation of small cells (see the white colored area in Fig. 1). This makes it difficult to obtain the analytical mean sojourn time in an MoA. Therefore, prior studies of analytical performance evaluation assumed that the mean sojourn time in an MoA should be estimated from the network trace data in advance [6]–[8]. One major limitation of this assumption is that the location update frequency in cellular systems is not fast enough because of signaling overhead concerns, thus resulting in less accurate estimation [8], [12]. Furthermore, collecting trace data for various deployment scenarios requires considerable experimental efforts.

In this letter, we propose a novel analytical method for estimating the mean sojourn time in an MoA of a two-tier heterogeneous network. The proposed method complements the limitation of existing evaluation frameworks [6]–[8]: we can integrate the proposed mean sojourn time analysis with existing frameworks, thereby obviating the need for acquiring trace-based sojourn time datasets. To the best of our knowledge, none of the prior studies have attempted to build such an

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analytical model for the mean sojourn time in an MoA under consideration.

For analytical tractability, our estimation method approximates a macro cell to a discrete grid consisting of small-cell-sized hexagons. Then, some hexagons are randomly selected to be ScA hexagons. Others remain MoA hexagons. In this way, we can approximate the heterogeneous network with small cells to the hexagonal grid. User mobility is also approximated as a discrete random jump on the grid. By applying the result of *stochastic geometry* [13] to the approximated model, we can estimate the analytical mean sojourn time in an MoA (see Section II). We performed an extensive set of simulations to validate the correctness of the proposed estimation method. The results suggest that the proposed method has excellent accuracy under the general random walk mobility models, including random waypoint, Brownian motion, tailored Brownian motion, and truncated Levy walk (see Section III).

II. Proposed Modeling and Estimation

A. System Model and Assumptions

We consider a two-tier heterogeneous macro/small cellular system where a macro cell includes multiple underlying small cells. The proposed model has the following assumptions:

- Cell deployment follows the widely used assumption as in [6]–[8], [12]. The radii of the macro and small cells are denoted by R_m and R_s, respectively. A macro cell includes N small cells within its coverage. The small cells are irregularly deployed.
- Mobile users move according to the general random walk mobility model defined in [14]. In a 2-dimensional field, each mobile user makes a random walk, expressed by a sequence of stochastic processes $\mathcal{S}$ as

$$\mathcal{S} = (L, \Theta, T_f, T_p) \tag{1}$$

where a mobile user makes a flight followed by a pause. L (> 0) is the length of the flight, Θ is the direction change from the previous flight, T_f (> 0) is the flight time and T_p (≥ 0) is the pause time. For each step, a mobile user chooses L, Θ, T_f and T_p from the probability density function $f_L(l)$, $f_\Theta(\theta)$, $f_{T_f}(t)$, and $f_{T_p}(t)$, respectively.[1] In this definition, the mobile velocity is expressed as $V = L/T_f$. The point at which each step begins is referred to the *waypoint*. The trajectory of each mobile user is expressed as a set of waypoints and lines generated by $\mathcal{S}$. Fig. 1 shows an example of a user trajectory consisting of five steps, each of which has length, that is, L_0, L_1, L_2, L_3, and L_4, respectively.

B. Mean Cell Sojourn Time in ScA

To calculate the mean sojourn time in an ScA, existing works can be adopted; note that our focus is on calculating the mean sojourn time in the MoA. In this subsection, we briefly introduce one of the well-known models. According to [11], the mean sojourn time in a cell with radius R_s can be calculated from the spatial distribution of mobile users as

$$E[S_s] = (E[T_f] + E[T_p]) \cdot \int_0^{2\pi} \int_0^{R_s} \widetilde{f}(r, \theta) \cdot dr \cdot d\theta \tag{2}$$

where $\widetilde{f}(r, \theta)$ is the spatial user density function for the incremental space $dA(r, \theta)$ around the point (r, θ) in the polar coordinate system. $\widetilde{f}(r, \theta)$ can be derived from $f_L(l)$, $f_{T_f}(t)$ and $f_{T_p}(t)$. For more details, please refer to [11], [15].

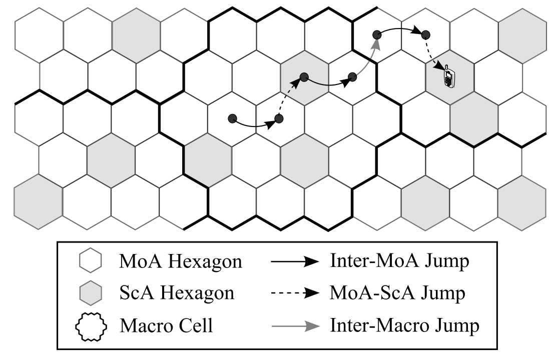


Fig. 2. Example of discrete grid approximation of a two-tier heterogeneous network.

C. Mean Cell Sojourn Time in MoA

As we discussed earlier, the irregular deployment of small cells makes it difficult to directly derive the mean sojourn time in an MoA (e.g., S_m in Fig. 1). To make it tractable, we approximate the network model in Section II-A as a hexagonal grid. We divide a macro cell into small-cell-sized hexagons, each of which has the size πR_s^2. Among them, N hexagons are randomly selected as the ScA hexagons. Fig. 2 illustrates an example of the proposed grid. The MoA and ScA hexagons are indicated in white and gray, respectively. As a consequence, the contiguous space random walk is also approximated to random *inter-hexagon* jumps, which can be categorized into three types: *inter-MoA*, *MoA-ScA*, and *inter-macro* jumps. Since small-cell-sized hexagons are used, the mean occurrence time of *inter-hexagon* jump can be approximated to $E[S_s]$.

Let us introduce the *length intensity* which indicates the probability that an arbitrary trajectory encounters the boundary of a *Poisson Voronoi tessellation* [13].[2] Let p_h, p_s, and p_m denote the *length intensity* of the *inter-hexagon*, *MoA-ScA*, and *inter-macro* boundaries, respectively. From *Remark 1* in [13], we obtain $p_h = 2\sqrt{\frac{1}{\pi R_s^2}}$ and $p_m = 2\sqrt{\frac{1}{\pi R_m^2}}$. And, we have $p_s \approx \gamma \cdot p_h$ where $\gamma (= \frac{N\pi R_s^2}{\pi R_m^2})$ denotes the proportion of ScA hexagons to every hexagon.

Now, the probability of every *inter-hexagon* jump is $p_h + p_m - p_h \cdot p_m$. Note that the *inclusive-exclusive principle* is used because some *MoA-ScA* boundaries are overlapped with *inter-macro* boundaries. Similarly, the probability that considers only the *MoA-ScA* and *inter-macro* jumps is

[1] From now on, we denote the probability density function and the cumulative density function of a random variable X as $f_X(\cdot)$ and $F_X(\cdot)$ respectively, for convenience.

[2] Our approximated network model is also a Poisson Voronoi tessellation where center-points are triangularly deployed.

TABLE I
MOBILITY PARAMETERS

Mobility Model	$f_L(l)$	$f_\Theta(\theta)$
RWP [11]	$2\lambda\pi l e^{-\lambda\pi l^2}$	$\begin{cases} \frac{1}{2\pi} & ,0 \le \theta \le 2\pi \\ 0 & ,otherwise \end{cases}$
BM [9]	$\frac{\frac{1}{t_f\sigma_v}\phi(\frac{l-t_f v_m}{t_f\sigma_v})}{\Phi(\frac{v_{max}-v_m}{\sigma_v})-\Phi(\frac{v_{min}-v_m}{\sigma_v})}$	$\begin{cases} \frac{1}{2\pi} & ,0 \le \theta \le 2\pi \\ 0 & ,otherwise \end{cases}$
TBM [9]	$\frac{\frac{1}{t_f\sigma_v}\phi(\frac{l-t_f v_m}{t_f\sigma_v})}{\Phi(\frac{v_{max}-v_m}{\sigma_v})-\Phi(\frac{v_{min}-v_m}{\sigma_v})}$	$\begin{cases} \frac{1}{\pi} & ,-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \\ 0 & ,otherwise \end{cases}$
TLW [14]	$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ixl-\lvert cl\rvert^\alpha} dx$	$\begin{cases} \frac{1}{2\pi} & ,0 \le \theta \le 2\pi \\ 0 & ,otherwise \end{cases}$

$p_s + p_m - p_s \cdot p_m = \gamma \cdot p_h + p_m - \gamma \cdot p_h \cdot p_m$. Consequently, we can express the probability that a mobile user encounters an ScA hexagon or inter-macro boundary for each *inter-hexagon* jump as

$$p = \frac{\gamma \cdot p_h + p_m - \gamma \cdot p_h \cdot p_m}{p_h + p_m - p_h \cdot p_m} = \frac{\gamma R_m\sqrt{\pi} + R_s\sqrt{\pi} - 2\gamma}{R_m\sqrt{\pi} + R_s\sqrt{\pi} - 2}$$
$$= \frac{N R_m R_s^2\sqrt{\pi} + R_m^2 R_s\sqrt{\pi} - 2N R_s^2}{R_m^3\sqrt{\pi} + R_m^2 R_s\sqrt{\pi} - 2R_m^2}. \quad (3)$$

From the above result, $(1-p)^K \cdot p$ is the probability that a mobile user takes $K-$consecutive *inter-MoA* jumps before encountering an ScA hexagon or inter-macro boundary. We derive its expectation as

$$E[K] = \sum_{k=1}^{\infty} k \cdot (1-p)^k \cdot p = \frac{1-p}{p}$$
$$= \frac{(1-\gamma)\cdot(R_m\sqrt{\pi}-2)}{\gamma R_m\sqrt{\pi} + R_s\sqrt{\pi} - 2\gamma}. \quad (4)$$

Since the mean occurrence time of an *inter-hexagon* jump is $E[S_s]$, $E[K]$ is the proportion of $E[S_m]$ to $E[S_s]$. By multiplying $E[S_s]$ by $E[K]$, we obtain the approximated mean sojourn time in the MoA as

$$E[S_m] \approx E[K] \cdot E[S_s] = \frac{(1-\gamma)\cdot(R_m\sqrt{\pi}-2)\cdot E[S_s]}{\gamma R_m\sqrt{\pi} + R_s\sqrt{\pi} - 2\gamma}. \quad (5)$$

III. VALIDATION RESULTS

We conducted an extensive set of simulations to validate the proposed estimation method. The general random walk mobility simulation was implemented using OMNeT++, an open discrete event driven simulator.[3] We also imported a GNU scientific library to generate the random walk process.[4]

We considered random waypoint (RWP) [11], Brownian motion (BM) [9], tailored Brownian motion (TBM) [9] and truncated Levy walk (TLW) [14] mobility models. These models belong to the class of general random walk models. We list their $f_L(l)$ and $f_\Theta(\theta)$ in Table I. In the table, v_{min}, v_{max} and v_m denote the min, max, and mean velocity of mobile users, respectively. $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability and cumulative density functions of standard normal distribution, respectively. BM and TBM use constant flight time, i.e., $T_f = t_f$. On the other hand, RWP and TLW generate the flight times as conditional random variables $T_f = \frac{L}{v_m}$ and $T_f = k \cdot L^{1-\rho}$, respectively. For the pause time T_p, RWP, BM and TBM use constant time t_p. In TLW, T_p is a Levy distributed random variable with the following density function:

$$f_{T_p}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ixt-\lvert ct\rvert^\beta} dx. \quad (6)$$

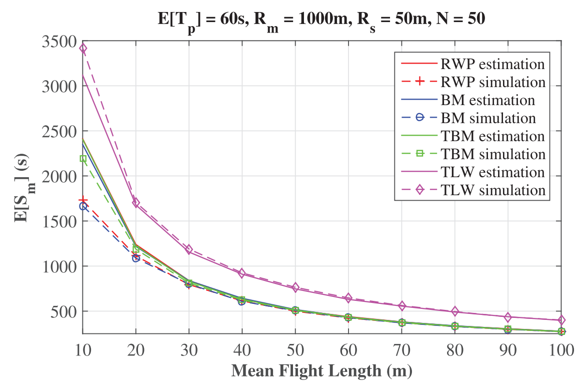
For more details regarding mobility parameters, please refer to [9], [11], and [14]. Unless otherwise mentioned, we followed the default settings listed in Table II.

To validate the correctness of the proposed estimation method, we measured $E[S_m]$ while $E[L]$ was increased from 10 m to 100 m in increments of 10 m. Mobility parameters were properly calibrated to set $E[L]$ as designated. Fig. 3a presents a comparison of estimated and measured mean sojourn times in

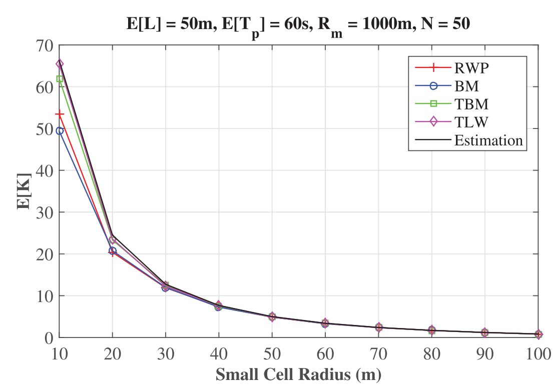
[3] http://omnetpp.org
[4] http://www.gnu.org/software/gsl/

TABLE II
SIMULATION PARAMETERS

Parameter Name	Value
Number of small cells (N)	50
Macro cell radius (R_m)	1000m
Small cell radius (R_s)	10m, 20m, ..., 100m (default: 50m)
Mean flight length ($E[L]$)	10m, 20m, ..., 100m (default: 50m)
Mean pause time ($E[T_p]$)	10s, 20s, 60s (default), 120s, 300s
Mean velocity (v_m)	2m/s, 5m/s, 10m/s (default), 20m/s, 50m/s
Simulation time	24h



(a) $E[S_m]$ with varying $E[L]$



(b) $E[K]$ with varying R_s

Fig. 3. Comparison between estimation and simulation results.

TABLE III
$E[K]$ WITH VARYING $E[T_p]$ AND v_m

$E[T_p]$	10*s*	20*s*	60*s*	120*s*	300*s*
RWP [11]	4.848	4.896	4.879	4.917	4.711
BM [9]	4.873	4.951	4.875	4.685	4.609
TBM [9]	4.975	4.949	4.946	4.916	4.840
TLW [14]	4.927	5.022	4.885	4.920	4.798
Estimation	4.998	4.998	4.998	4.998	4.998

v_m	2m/s	5m/s	10m/s	20m/s	50m/s
RWP [11]	4.836	4.865	4.879	4.980	4.926
BM [9]	3.275	4.663	4.875	4.936	4.969
TBM [9]	4.394	4.875	4.946	4.976	5.009
Estimation	4.998	4.998	4.998	4.998	4.998

the MoA. The average differences of RWP, BM, TBM and TLW are 6.017%, 6.115%, 2.175%, and 2.321%, respectively. The estimated and measured results generally fit well to each other. The estimation differences of RWP and BM are more than 10% when $E[L] \leq 20$ m. This is because RWP and BM have zigzag (or back-and-forth) movement patterns. With short flight distances, zigzag movements between cell boundaries result in a large number of short sojourn time samples, and thus mean values are biased. In contrast, fewer zigzag movements appear in TBM and TLW. TBM does not produce a sharp curved trajectory (see $f_\Theta(\theta)$ in Table I). TLW generates frequent long distance flights owing to the nature of the Levy distribution (see $f_L(l)$ in Table I). Their results remain plausible at small flight distances. For example, their estimation differences are less than 10% at $E[L] = 10$ m.

Next, we considered how the size of a small cell area affects the estimation accuracy. We plot $E[K]$ instead of $E[S_m]$ for the sake of illustration. As illustrated in Fig. 3b, the average estimation difference of RWP, BM, TBM and TLW is 5.389%, 6.001%, 1.779%, and 1.564%, respectively. Similar to the previous scenario, the proposed method generally shows excellent estimation capabilities. When $R_s \leq 20$ m, the estimation differences of RWP and BM increase to approximately 25%. This is attributed to a well-known phenomenon called the *biased sampling problem* [16]. For example, suppose a zigzag moving user wanders in the middle of the MoA when the small cells are sparsely deployed. Because the user is not likely to encounter an inter-tier or inter-macro cell boundary, the users sojourn time is not likely to be sampled by the simulation. Due to such hidden long sojourn time users, the measured sojourn time is biased, thus $E[K]$ is also. In the cases of TBM and TLW, thanks to fewer zigzag movements, their estimation differences remain low (6.512% and 1.294% for TBM and TLW, respectively, at $R_s = 10$ m).

We also conducted the simulations with varying $E[T_p]$ (10 s $\sim$ 300 s) and v_m (2 m/s $\sim$ 50 m/s), and observed that their average estimation differences were less than 10%. We present brief results in Table III instead of plots for space reasons.[5] The simulation results suggest that the proposed estimation method has excellent accuracy, especially for the Levy walk, which is a recently emerged model for human mobility studies.

[5]TLW is not considered in varying v_m because the velocity of the TLW is defined as a conditional random variable $V = L^\rho / k$.

IV. CONCLUSION

In this letter, we propose a simple but powerful method for estimating the mean sojourn time in macro-cell-only areas of two-tier heterogeneous networks where macro-embedded small cells are irregularly deployed. By approximating the user mobility in a continuous space as discrete random jumps on a hexagonal grid, we develop a tractable analytical procedure to derive the proportion of the mean sojourn time in the macro-cell-only area to the small-cell-covered area. The simulation results show that the proposed method has plausible accuracy for general random walks, including the random waypoint, Brownian motion, tailored Brownian motion and truncated Levy walk models. This implies mobility-model-free application of our analysis. As a new complementary method, the proposed estimation technique can be integrated into existing performance evaluation frameworks [6]–[8] considering macro/small cell heterogeneous networks.

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